Viscous accretion disks with shocks

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Introduction and Motivation

The accretion of matter onto compact objects is the fundamental mechanism powering a variety of high energy astrophysical sources, such as X-ray binaries and active galactic nuclei.Gravitational energy released during accretion has regularly been invoked to explain observed luminosities of these astrophysical sources, their spectral states, and the formation of jets or outflows around these objects. Since black holes have neither hard surface, nor intrinsic atmosphere these jets or outflows have to originate from the accreting material itself. And observationally it has been seen that the jets and outflows originate from regions close to the event horizon, especially Junor et. al. (1999, Nature) showed that the M87 jet originates within a region of 30-100r_s around the central object. The accretion disc models on the other hand varies from purely Keplerian discs to models having strong advection, and sometimes combination of all these models. Of all these models the shock in accretion model satisfies this criteria, since shocks occur typically in regions few tens of r_s away from the black hole. Here we present two extreme cases for the jet formation, one is thermally driven jet due to adiabatic shock waves and other is by the acceleration of energetic particles due to isothermal shock waves. From these two cases we quantify the jet solutions.

Basic equations

Radial momentum equation:

$$u\frac{du}{dx}+\frac{1}{\rho}\frac{dp}{dx}+\frac{1}{2(x-1)^2}-\frac{\lambda(x)^2}{x^3}=0.$$

Mass flux equation:

$$\dot{M}=4\pi\rho hux.$$

Angular momentum distribution equation:

$$u\frac{d\lambda(x)}{dx}+\frac{1}{\rho hx}\frac{d(x^2t_{x\phi})}{dx}=0.$$

Entropy generation equation:

$$\rho huT\frac{ds}{dx}=Q^+-Q^-.$$

Where, The local variables u, a, x, p, ρ and λ in the equations are the radial bulk velocity, sound speed, radial distance, isotropic pressure, mass density and specific angular momentum of the flow, respectively. Here $t_{x\phi}$ is the viscous stress tensor, s is entropy density of the flow, and T is the local temperature. Q^+ and Q^- are the heat gained and lost by the flow, respectively. Here we are considering only viscous heating and ignoring cooling.

Here, $Q^+=\frac{t_{x\phi}^2}{\eta}$, and $t_{x\phi}=\eta x\frac{d\Omega}{dx}$. Where, α is Shakura-Sunyaev viscosity parameter, $\eta=\rho\nu h$ is viscosity coefficient or dynamic viscosity, $\nu=\frac{\alpha a^2}{\Omega_k}$ is kinematic viscosity, and Ω_k is Keplerian angular frequency. Considering hydrostatic equilibrium in vertical direction, the local disc height is obtained as: $h(x)=\sqrt{\frac{2}{\gamma}}ax^{1/2}(x-1)$. Where a is adiabatic sound speed defined as $a=\sqrt{\frac{\gamma p}{\rho}}$, γ is adiabatic index.

equations of motions for outflows

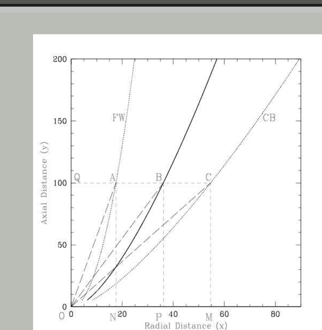

fig.1 Jet geometry for $\lambda=1.75$. In figure $OA=r_{FW}, OC=r_{CB}, QC=x_{CB}, CM=y_{CB}; CM=BP(y_i)=AN(y_{FW}); QA=x_{FW}; QB=x_j; AndOB=r_j$. FW and CB are marked in the figure.(Chattopadhyay, I.; Das, S.,2007NewA.12.454C)

Numerical simulations by Molteni et al. (1996) suggests that the outflowing matter tends to emerge out between two surfaces namely, the funnel wall(FW) and centrifugal barrier(CB). In Fig.1, the schematic diagram of the jet geometry is shown. The centrifugal barrier(CB) surface is defined as the pressure maxima surface and is expressed as

$$x_{CB}=[2\lambda^2r_{CB}(r_{CB}-1)^2]^{\frac{1}{4}},$$

where, $r_{CB}=\sqrt{x_{CB}^2+y_{CB}^2}$, spherical radius of CB. Here x_{CB}, y_{CB} are the cylindrical radius and axial coordinate (i.e., height at r_{CB}) of CB. We compute the jet geometry with respect to y_{CB} i.e.,$y_{FW}=y_j=y_{CB}$, where y_{FW} and y_j are the height of FW and the jet at r_{CB}, respectively.

$$x_{FW}^2=\lambda^2\frac{(\lambda^2-2)+\sqrt{(\lambda^2-2)-4(1-y_{CB}^2)}}{2},$$

where, x_{FW} is the cylindrical radius of FW. We define the cylindrical radius of the outflow

$$x_j=\frac{x_{FW}+x_{CB}}{2}.$$

The spherical radius of the jet is given by $r_j=\sqrt{x_j^2+y_j^2}$. In Fig.1, OB($=r_j$) defines the streamline (solid) of the outflow. The total area function of the jet is obtained as,

$$\mathcal{A}=2\pi(x_{CB}^2-x_{FW}^2).$$

The integrated momentum balance equation for jet is given by,

$$\mathcal{E}_j=\frac{1}{2}v_j^2+na_j^2+\frac{\lambda_j^2}{2x_j^2}-\frac{1}{2(r_j-1)},$$

where $\mathcal{E}_j$ and λ_j are the specific energy and the angular momentum of the jet, respectively. The integrated continuity equation is,

$$\dot{M}_{out}=\rho_j v_j\mathcal{A}.$$

Calculation of shock locations:

Physical quantities, which is conserved at the shock:

- Mass flux

$$\dot{M}_+=\dot{M}_-.$$

- In presence of mass loss, the condition for conservation of mass flux takes the following form,

$$\dot{M}_+=\dot{M}_--\dot{M}_{out}=\dot{M}_-(1-R_{\dot{m}}).$$

Where, The expression for $R_{\dot{m}}$ is calculated by assuming that jets are launched with the same density as the post-shock flow and is given by,

$$R_{\dot{m}}=\frac{\dot{M}_{out}}{\dot{M}_-}.$$

- Momentum flux

$$p_++\rho_+u_+^2=p_-+\rho_-u_-^2.$$

- Angular momentum flux

$$\dot{J}_+=\dot{J}_-.$$

Where, $\dot{J}$ expressed as: $\dot{J}=\dot{M}x^2\Omega-G$. Here, subscripts minus(-) and plus(+) denote the quantities before and after the shock.
For Rankine-Hugoniot shock condition, energy flux is conserved at the shock, which is expressed as:

$$\dot{E}=\dot{M}[\frac{u^2}{2}+\frac{a^2}{\gamma-1}-\frac{\lambda^2}{2x^2}+\frac{\lambda_0\lambda}{x^2}+\Phi].$$

At the shock location it is written as:

$$\dot{E}_+=\dot{E}_-.$$

And for isothermal shock condition, sound speed is conserved at the shock. Therefore,

$$a_+=a_-.$$

With the help of mass flux, momentum flux and angular momentum flux conservation equations at the shock, we get local flow variables of supersonic branch.

Shocks parameter space

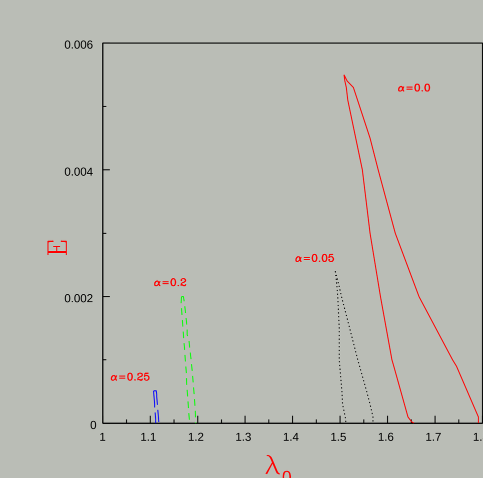

Fig.2 Representation of adiabatic shock parameter space spanned by the specific angular momentum at horizon λ_0 and the energy E with Shakura-Sunyaev viscosity parameter $\alpha=0.0, 0.05, 0.2$ and 0.25 when $\gamma=1.4$.

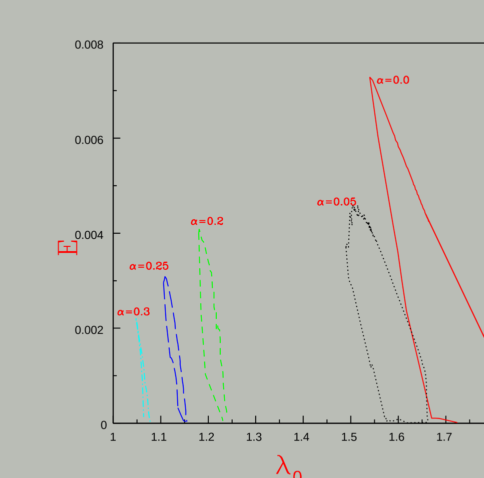

Fig.3 This figure represents the isothermal shock parameter space again spanned by the specific angular momentum at horizon λ_0 and the energy E with $\alpha=0.0, 0.05, 0.2, 0.25$ and 0.3 when adiabatic index $\gamma=1.4$.

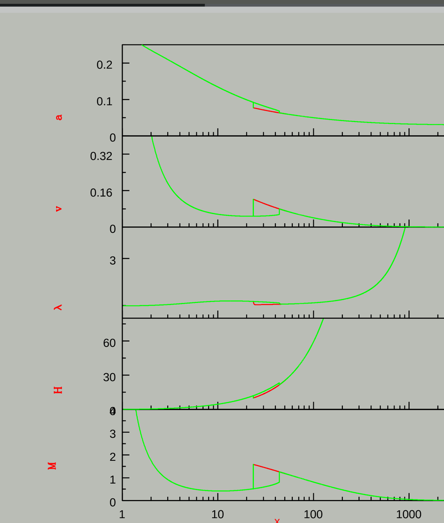

Fig.4 Represents one of adiabatic shock solution with mass loss and without mass loss when the specific angular momentum at horizon $\lambda_0=1.495$, Shakura-Sunyaev viscosity parameter $\alpha=0.05$ and the input energy $E=0.00215$.

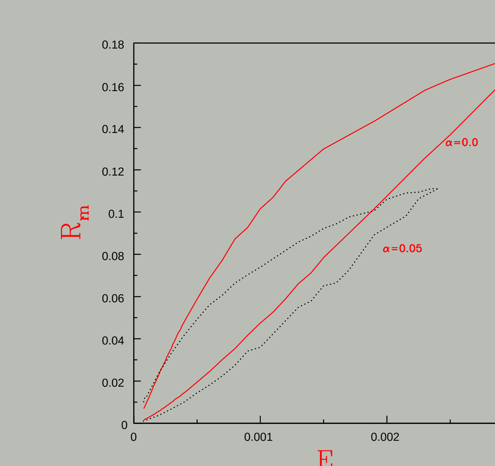

Fig.5 Shows regions of the mass outflow rate at the shock with the variations of energy E and anguler momentum at horizon λ_0 for Shakura-Sunyaev viscosity parameter $\alpha=0.0, 0.05$ when $\gamma=1.4$.

conclusions

- We found that isothermal shock can form comparatively at higher viscosity parameter then adiabatic shock.
- In case of mass loss the shock location shifted towards center of BHs form the shock location of without mass loss shock.
- The mass outflow rate is higher in inviscid with comparison of viscid case and mass outflow rate decreases with the lowering in incident energy.

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