Richardson-Lucy Deconvolution Technique for the CZT detector Array of ASTROSAT

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0.1 Introduction

The CZT imager aboard ASTROSAT is capable of studying X-rays in high (20 KeV to 100 KeV) and the low (5 KeV to 20 KeV) energy range. To make it capable for imaging in these energy ranges appropriate collimators for respective energies have been designed. Initial attempts to image the sky were made by following the standard cross correlation technique. The results of which were reported and presented in the BDR.

Due to the introduction of the on board calibration unit the mask pattern casts partial shadows on the detector. The standard cross correlation technique fails in such cases and hence we look for other imaging techniques to improve on the imaging quality. The report describes a alternate imaging technique to improve the imaging with the CZT.

0.2 Problems in the Cross correlation technique

Each quadrant of the CZT imager is divided into basic units by the High energy collimators. To facilitate the low energy imaging each unit is further divided into 8 parts by the low energy collimators. The Collimator units are placed above the on board calibration unit of height 100cm. The on board calibration unit does not have any collimators in it and hence the photons that are cleared at the base of the collimator unit are free to pass to the detector without further collimation. The absence of collimators in the calibration unit results in seepage of photons into adjacent pixels producing partial shadowing effects. The cross correlation technique which uses the basic mask pattern as the correlation input with the detected shadow, does not accurately model the response of the camera. We therefore look for a alternative technique where the expected shadows for sources at different angles would be the primary input rather than the mask pattern it self. One such algorithm is the Richardson-Lucy algorithm which we have attempted to implement.

0.3 Justification for the choice of the Richardson-Lucy technique

Requirements of the imaging technique

1.As a result of the introduction of the calibration unit full quadrant imaging should be done instead of imaging a single basic unit. The Richardson -Lucy algorithm facilitates the full quadrant imaging. Further the strength of the source obtained by Richardson-Lucy algorithm was not reliable as the technique operates only on area normalized response functions. To obtain the right strength of the source we adopt the minimum chi-square fit to the full observed shadow, with sources located at positions reported by the Richardson-Lucy algorithm. The results of the minimum chi-square fit done using the SVD (Singular Value Decomposition) method yields accurate results for the source strength. Thus by following the two step procedure we obtain the exact source position with accurate source strength which is the main objective of the imaging algorithm.

Note : The minimum chi-square fitting is a alternative to the Maximum likelihood algorithm which will eventually replace it.

0.4 Richardson-Lucy Deconvolution Algorithm

The requirements for the Richardson-Lucy algorithm are the shadow patterns and the spatially coded detector data. The spatially coded detector data is obtained by Monte Carlo simulation. The shadow of the mask pattern for all possible sky locations in the field of view of the camera are generated and stored for both the high and the low energies to be used in the Richardson-Lucy algorithm.

With the result of the simulation and the shadows obtained the Richardson-Lucy algorithm originally defined in Binney. J & Merrifield. M ,1998 to be used with PSF is modified so that the algorithm can be used with shadow patterns.

The first step in the Richardson-Lucy algorithm is to area normalize the detector data and the shadow patterns and store the area normalized shadow patterns. We also find the total number of open elements in the shadow patterns which is used to area normalize the shadow patterns and later used as the scaling factor in obtaining the source strength.

We assume a flat source distribution initially (i.e,. sources to be present in all the sky locations) and this source distribution is area normalized.

Using the area normalized shadow patterns and the source distribution one generates a prediction for the possible photon count distribution.

The detector data for a simulated source is obtained and area normalized to unity. The initial predicted distribution for the detector is used along with the area normalized detector count to obtain a correction factor for the source distribution. The initial guess is now changed by this correction factor.

The corrected source distribution is now used to obtain a prediction for source distribution. The iterative procedure continues until there is no further improvement possible in the source distribution or the difference in the correction factor is less than a given threshold. The resultant source distribution obtained will be the actual source distribution of the sky for the simulated source but has to be scaled by the factor used to area normalize the shadows and the detector counts.

The final source distribution is obtained by dividing the source distribution array by the scaling factor or the number of open elements for the source simulated and then multiplying it by the total number of detected photons.

The result due to the Lucy algorithm was accurate for position but did not yield the right values for source strength. To obtain the right source strength the result of the Lucy algorithm is fed into the SVD fit algorithm and the resulting matrix gives the accurate value for the source strength at the position as generated by the Lucy algorithm. Mathematical representation of the Richardson-Lucy algorithm is given below.

A step wise representation of the Lucy algorithm is given below

(a). The photon count distribution is predicted using the following relation:

$$P_{i,j} = \sum_{i,j} \sum_{k,m} R_{ijkm} \times S_{ij}$$

$$\forall\, 0 \le i \le ndetx \;\forall\, 0 \le j \le ndety$$

$$\forall\, 0 \le k \le nskyx \;\forall\, 0 \le m \le nskyy$$

where S_{ij} is the area normalized source distribution in the sky.

R_{ijkm} are the shadow patterns. *ndetx* and *ndety* are number of detector elements in the X direction and in the Y direction respectively.*ndetx* and *ndety* equal to 67 in both the high energy and the low energy case.
nskyx and *nskyy* are the number of sky elements in the X and the Y direction respectively. For high energy *nskyx* is equal to *nskyy* which is 37 and for low energy case *nskyx* is equal to 9 and *nskyy* is 19.
(b). The predicted values of the photon counts are used to obtain the correction factors C_{ij} for every sky pixel using the following relation:

$$C_{ij} = \sum_{k,m} \sum_{i,j} \frac{R_{ijkm} \times D_{ij}}{P_{ij}}$$

where D_{ij} are the area normalized simulated photon counts.
(c). The values of the source distribution S_{ij} are corrected using the correction factors C_{ij} as follows:

$$S_{ij} = S_{ij} \times C_{ij} \qquad \forall\, 0 \leq i \leq nskyx \; \forall\, 0 \leq j \leq nskyy$$

The corrected source distribution S_{ij} is used to obtain the next set of predictions of photon count distribution P_{ij} as in (a). At each iterative step the maximum(*max*) of ($S_{i,j}$ - C_{ij}) is calculated. The iterative procedure consisting of steps (a), (b) and (c) is carried on till the difference of (1 - *max*) obtained is less than a specified tolerance limit (T).

0.5 Minimum Chi-square Fit

The source strength which are obtained as a result of the Richardson-Lucy as described earlier are not accurate as the Richardson-Lucy algorithm operates on area normalized functions. Hence to obtain the right source strengths as the next step we generate a minimum chi-square fit to the observed shadows as a linear combination of shadows for sources in the sky locations reported by the Richardson-Lucy algorithm. A root mean square (RMS) of the source distribution which is the result of the Richardson-Lucy algorithm is found. All source strengths above (2 × RMS) are considered as possible source positions. A function that obtains the shadow pattern and unwraps it into a single dimension array is developed. This function is fed as one of the inputs to the SVD fit algorithm.

Let N be the number of sources reported by the Richardson-Lucy algorithm. We extract the unwrapped shadows corresponding to the reported sources. For each of the shadows we obtain the prediction array by scaling the shadow with the corresponding source strength

$$P_{ik} = R_{ik} \times S_i \qquad (i = 1, N)$$

where S_i is the source strength as generated by Richardson-Lucy algorithm.
R_{ik} are the unwrapped shadow patterns.

Chi-square is given by the relation

$$\chi^2 = \sum_k \left(\sum_i P_{ik} - D_k \right)^2 \qquad (1)$$

where D_k is the unwrapped detected source counts, the result of simulation.

To minimize the chi-square the derivative of equation (1) with respect to S_i is obtained and equated to zero.

$$\frac{\partial \chi^2}{\partial S_i} = 2 \sum_k \left(\sum_j (P_{jk} - D_k) \right) \frac{\partial P_{ik}}{\partial S_i} = 0, \qquad i = 1, N$$

The solution to this set of equations yields the source strengths S_i. The unwrapped detector data along with the function that generates the unwrapped shadow patterns is fed to the SVD algorithm. The SVD algorithm returns a reduced chi square fit for the shadows and the accurate source strengths are obtained. These source strengths are then replaced in the positions generated by Richardson-Lucy algorithm to obtain the final image.

0.6 Results

In this section a few results of the reconstructed sky using the Richardson- Lucy algorithm are presented. We have successfully reconstructed the sky for both the high as well as the low energy cases. The results shown will also include reconstructing the sky for not only one source simulated in the field of view of the camera but also for more than one source simulated in the field of the camera.

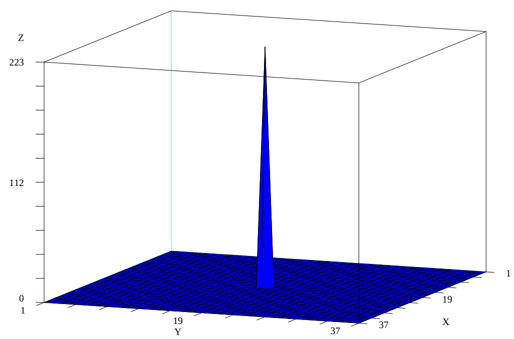

Figure 1: Reconstructed sky for source at normal

Figure 1 shows the reconstructed sky for high energy. Source is simulated at normal . The total number of photons simulated is 10,00,000. The flux is in units of counts/maskelement which corresponds the 223 counts/maskelement. The source strength recovered is 223.20 counts/mask element. The threshold limit T for the convergence of the Richardson-Lucy algorithm is 0.0001.

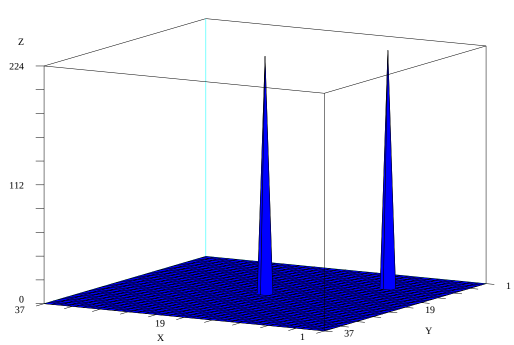

Figure 2: Reconstructed sky with two sources in field of view

Figure 2 shows the reconstructed sky for high energy case with two sources in the field of view. Sources are separately simulated one at normal incidence and another source at an angle of incidence equal to (2.8623°, 2.8623°). The total number of photons simulated is 10,00,000 in both cases and the two simulated files are added pixel by pixel to be fed as input to the Richardson-Lucy algorithm. The threshold T for the the Richardson-Lucy algorithm is 0.0001. The source strengths recovered are 223.33counts/ maskelement and 223.71 counts/maskelement respectively.

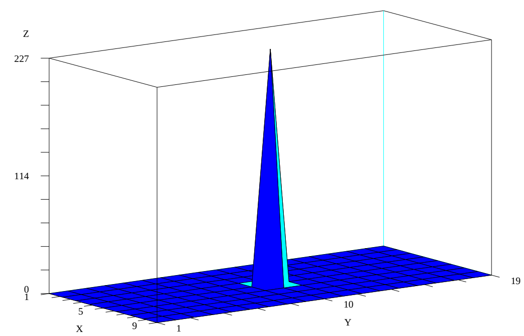

Figure 3: Reconstructed sky for source at normal

Figure 3 shows the reconstructed sky for low energy. Source is simulated at normal . The total number of photons simulated is 10,00,000 and the threshold T for the Richardson-Lucy algorithm is 0.0001. The source strength recovered is 227.05 counts/maskelement.

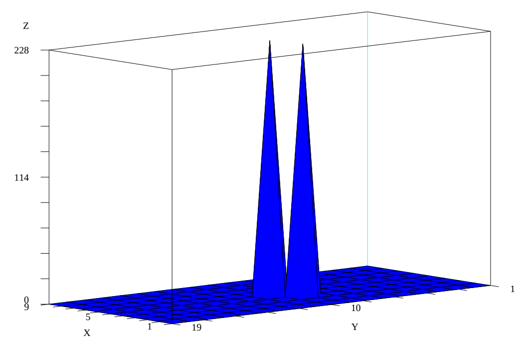

Figure 4: Reconstructed sky with two sources in field of view

Figure 4 shows the reconstructed sky for low energy case with two sources in the field of view. Sources are simulated separately one at normal incidence and other at an angle of incidence equal to (0.2865 °, 0.2865 °). The total number of photons simulated is 10,00,000 in both cases and the two simulated files are added pixel by pixel to be fed as input to the Richardson-Lucy algorithm. The threshold for the convergence T is equal to 0.0001. The source strengths recovered are 228.14 counts/maskelement and 225. 58 counts/maskelement respectively.