Coordinate Transformation Conventions for ASTROSAT CZT Imager

Dipankar Bhattacharya

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Introduction

This document summarizes the coordinate transformation conventions being used in CZTI software to connect the location of sources in the Camera Coordinate System (CCS) with their Right Ascension (RA) and Declination (DEC) values defined in the Inertial Coordinate System (ICS).

System definitions and basic transformations

The specification of coordinates in the CCS will normally be made as the tangent plane angles θ_x and θ_y. These angles are related to the components of the CCS unit vector V to the source as:

$\Theta_x = \arctan (V_x / V_z)$
$\Theta_y = \arctan (V_y / V_z)$

i.e. $V_z = 1/\mathrm{sqrt}(1+\tan^2\theta_x + \tan^2\theta_y)$; $V_x = V_z \tan\theta_x$; $V_y = V_z \tan\theta_y$

This is a natural choice of representation as the shift of the coded mask shadow on the detector plane for an off-axis source is proportional to $\tan\theta_x$ and $\tan\theta_y$ in respective directions.

The orientation of the CZTI with respect to the satellite body is specified through a TELDEF file (kept in CALDB), which provides the matrix coefficients of transformation between the CCS and the satellite Body Coordinate System (BCS). This is called an **[ALIGNM]** matrix, to be used as

V_{BCS} = **[ALIGNM]** * V_{CCS}

During flight, the orientation of the satellite body with respect to the Inertial Coordinate System will be given by the satellite attitude. Specifically, ASTROSAT attitude files will provide the (RA, Dec) values of each of the satellite BCS principal axes, namely Yaw, Roll and Pitch, as (RA_y, DEC_y), (RA_r, DEC_r) and (RA_p, DEC_p) respectively.

Using this, it is simple to construct a transformation between the CCS and the ICS:

First, we evaluate the direction cosines of the three BCS axes in the ICS:

Ly = cos(RA_y)cos(DEC_y) ; My = sin(RA_y)cos(DEC_y) ; Ny = sin(DEC_y)
Lr = cos(RA_r)cos(DEC_r) ; Mr = sin(RA_r)cos(DEC_r) ; Nr = sin(DEC_r)
Lp = cos(RA_p)cos(DEC_p) ; Mp = sin(RA_p)cos(DEC_p) ; Np = sin(DEC_p)

The transformation from BCS to ICS, and hence CCS to ICS, is then

V_{ICS} = **[M]*** V_{BCS} = **[M]*** **[ALIGNM]** * V_{CCS}

Where the matrix **[M]** is defined as

$$[\mathbf{M}] = \begin{matrix} Ly & Lr & Lp \\ My & Mr & Mp \\ Ny & Nr & Np \end{matrix}$$

The inverse transformation, from ICS to CCS, is simply

V_{CCS} = **[ALIGNM]**^{T} * **[M]**^{T} * V_{ICS}

where the superscript "T" indicates transpose.

Once the CCS vector components are available, one can convert them to (θ_x , θ_y) using the relations mentioned above. The relation between the components in the ICS and (RA, DEC) are:

$V_{x,ICS}$ = cos(RA) cos(DEC); $V_{y,ICS}$ = sin(RA) cos(DEC); $V_{z,ICS}$ = sin(DEC)

i.e. DEC = arcsin($V_{z,ICS}$) ; RA = arctan2 ($V_{y,ICS}$, $V_{x,ICS}$) mod 360 deg

CZTI average aspect computation

The pipeline routine **`cztgaas`** computes the average aspect of the CZTI during a given observation. This routine reads the TELDEF file, Level-1 attitude file and the event file. It outputs the aspect array of CZTI in a file, and also writes the average aspect in the event file.

In the CALDB, a separate TELDEF file is provided for each CZTI quadrant. Therefore **`cztgaas`** treats each quadrant independently and outputs a separate aspect array file for each quadrant. Two CCS vectors (0,0,1) and (0,1,0) are transformed to the ICS using the method outlined above to yield unit vectors N (NX,NY,NZ) and Nt (NXt,NYt,NZt) in the ICS. The content written to the aspect array file consists of an UT timestamp and the vector components NX, NY, NZ, NXt, NYt, NZt in a row, with subsequent rows at intervals of 1 sec (or nearest possible interval higher than 1 sec), for the duration of time covered by the event file.

In order to obtain average attitude over the full pointing duration, the vector components are individually averaged over the duration to yield <NX>, <NY>, <NZ>, <NXt>, <NYt>, <NZt>. The two vectors are then normalized to give components (Ax=<NX>/Rn, Ay=<NY>/Rn, Az=<NZ>/Rn), where $Rn=(<NX>^2 + <NY>^2 + <NZ>^2)^{1/2}$ and (Tx=<NXt>/Rt, Ty=<NYt>/Rt, Tz=<NZt>/Rt) where $Rt=(<NXt>^2 + <NYt>^2 + <NZt>^2)^{1/2}$.

The average RA, Dec of the CZTI central pointing direction is then obtained as

DEC = arcsin(Az) ; RA = arctan2(Ay, Ax) mod 360 deg

And an average TWIST angle is computed from:

CT = Tz cos(DEC) – Tx sin(DEC)cos(RA) – Ty sin(DEC)sin(RA)
ST = Ty cos(RA) – Tx sin(RA)
TWIST = arctan2(ST, CT)

Where RA, DEC correspond to the central pointing direction derived above. The average RA, DEC and TWIST values are then written to the event file.

WCS keywords in CZTI image products

The values output by **`cztgaas`** are used in providing the World Coordinate System (WCS) transformation keywords in the image products generated by the **`cztimage`** module. The relationship is as follows:

The average RA, Dec values determined by **`cztgaas`** are entered as the values of the `CRVAL1` and `CRVAL2` keywords respectively, for the central pixel of the image. The negative of the TWIST angle is the value of the `CROTA1` and `CROTA2` keywords. The `CDi_j` matrix of WCS has entries

`CD1_1` = cos(TWIST)*pixel_pitch
`CD1_2` = -sin(TWIST)*pixel_pitch
`CD2_1` = sin(TWIST)*pixel_pitch
`CD2_2` = cos(TWIST)*pixel_pitch

where pixel_pitch is the increment in angle going from one pixel to the next at the centre of the field. It depends on the oversampling factor with which the image has been created:

pixel_pitch = arctan[pixel_width/(mask_height*oversamp)]

For CZTI, the average "pixel_width" is 2.4375 mm and mask_height is 477 mm. “oversamp” is the oversampling factor; in the default setting this has a value of 5, which corresponds to a pixel_pitch of 0.058557 deg, or 3.5134 arcmin.

The coordinate type keywords are entered as:

```
CTYPE1 = ‘RA---TAN’
CTYPE2 = ‘DEC--TAN’
```

since the image produced is in tangent plane coordinates.

Transformations using RA, DEC, TWIST

Once the pointing direction (RA0, DEC0) of the CZTI central axis, and the TWIST angle has been derived by **`cztgaas`,** these values can be used to provide the transformation between the CCS and the ICS without further recourse to the attitude file. The following expressions may be used in these transformations:

From the (RA, DEC) of a source to camera coordinates (θ_x , θ_y):

Vz = cos(DEC0)cos(DEC)cos(RA-RA0) + sin(DEC0)sin(DEC)
Vx = -cos(DEC)sin(RA-RA0)
Vy = -[sin(DEC0)cos(DEC)cos(RA-RA0) – cos(DEC0)sin(DEC)]

θ_x = arctan[{Vx cos(TWIST) + Vy sin(TWIST)}/Vz]
θ_y = arctan[{Vy cos(TWIST) – Vx sin(TWIST)}/Vz]

and from (θ_x , θ_y) to (RA, DEC) of the source:

Vz = 1/sqrt(1+$\tan^2\theta_x$+ $\tan^2\theta_y$) ; Vx = Vz $\tan\theta_x$; Vy = Vz $\tan\theta_y$
Xm = Vx cos(TWIST) – Vy sin(TWIST)
Ym = Vy cos(TWIST) + Vx sin(TWIST)

Zs = Vz sin(DEC0) + Ym cos(DEC0)
Xs = Vz cos(RA0)cos(DEC0) – Ym cos(RA0)sin(DEC0) + Xm sin(RA0)
Ys = Vz sin(RA0)cos(DEC0) – Ym sin(RA0)sin(DEC0) - Xm cos(RA0)

DEC = arcsin (Zs)
RA = arctan2 (Ys, Xs) mod 360 deg